Multi-Fidelity Neural Network Surrogate Modeling for Large-Scale Bayesian Inverse Problems with Applications to Inverse Acoustic Scattering by Random Domains

Bioli Ivan[1], Dölz Jürgen[2] and Henríquez Fernando[3]

[1]École Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland
[2]Institute for Numerical Simulation, University of Bonn, Friedrich-Hirzebruch-Allee 7, 53115 Bonn, Germany
[3]MCSS, Chair of Computational Mathematics and Simulation Science, École Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland

The computational approximation of large-scale Bayesian Inverse Problems (BIPs), i.e. characterized by a large number of available observations, involves utilizing techniques such as Markov Chain Monte Carlo (MCMC) to generate samples from the posterior distribution of the BIP. The primary obstacle in this approach arises from the necessity of repeatedly evaluating the high-fidelity forward model. This becomes cost-prohibitive for complex computational models, such as parametric PDEs with high-dimensional inputs, prompting the need for effective computational surrogates to facilitate MCMC execution.

In this work, we introduce a Multi-fidelity Neural Network (MF-NN) surrogate construction for Bayesian Inverse Problems (BIPs), inspired by the approach presented in [3]. Initially, an offline step involves constructing a low-fidelity prior-based surrogate solver using a neural network (NN) following the Galerkin-POD-NN technique [2]. Subsequently, a local online refinement process utilizes a NN to improve the low-fidelity ap- proximation by incorporating newly computed high-fidelity data. Unlike existing methods, the MF-NN approach enables an accurate approximation of the posterior distribution of the BIP by adaptively refining the model only where needed, specifically in regions where the posterior concentration phenomenon occurs. By focusing the computational effort only where necessary, the number of high-fidelity model evaluations is significantly reduced.

As a concrete application, we consider the time-harmonic acoustic scattering by random domains, as in [1]. Numerical experiments validate the effectiveness, efficiency and accuracy of the AMF-NN approach, especially in challenging settings for BIPs, including both the low noise and large data limits.

REFERENCES

[1] Dölz, J., Harbrecht, H., Jerez-Hanckes, C., and Multerer, M. (2022). Isogeometric multilevel quadrature for forward and inverse random acoustic scattering. Computer Methods in Applied Mechanics and Engineering 388, p. 114242.

[2] Hesthaven, J. S., and Ubbiali, S. (2018). Non-intrusive reduced order modeling of non-linear problems using neural networks. Journal of Computational Physics 363, pp. 55-78.

[3] Li, Y., Wang, Y., and Yan, L. (2023). Surrogate modeling for Bayesian inverse problems based on physics-informed neural networks. Journal of Computational Physics 475, p. 111841.