Wavenumber-robust deep ReLu neural network emulation in acoustic wave scattering

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We present wavenumber-robust error bounds using deep neural networks to emulate the solution to the time-harmonic, sound-soft acoustic scattering problem in the exterior of a smooth, convex, two-dimensional obstacle.

The starting point of our analysis is the reduction of the scattering problem in the unbounded exterior region to its (bounded) boundary by means of the wavenumber-robust Combined Field Integral Equation (CFIE), yielding a second-kind boundary integral equation posed on the smooth surface $\Gamma$ of the scatterer. This BIE is well-posed in $L^2(\Gamma)$, with explicit bounds on the continuity and stability constants that depend explicitly on the (non-dimensional) wavenumber $\kappa$.

Utilizing well-known wavenumber-explicit asymptotics of the solution to this problem, as introduced in the work of Melrose and Taylor \cite{MelroseTaylor}, we explore the numerical approximation of the BIE using fully connected, deep feed-forward neural networks (DNNs) with the Rectified Linear Unit (ReLU) as the chosen activation function \cite{ElbrachterPerekrestenkoGrohsBoelcskei}. The results presented here can be straightforwardly extended to different activation functions such as the hyperbolic tangent or the Rectified Power Unit.

Through a constructive argument, we prove the existence of DNNs affording an $\epsilon$-error in the $L^\infty(\Gamma)$-norm with a fixed and small width and a depth that increases spectrally with the accuracy $\epsilon$ and polynomially with respect to $\log(\kappa)$. By spectral accuracy, we mean that there exists $\alpha > 0$ such that for each $n \in \mathbb{N}$, there exists a constant $C_n > 0$, such that for a prescribed accuracy $\epsilon > 0$, the complexity of the DNN is bounded by $C_n \epsilon^\frac{2}{\alpha}$.

REFERENCES