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Wavenumber-robust deep ReLu neural network emulation in acoustic wave scattering

Henríquez Fernando[1] and Schwab Christoph[2]

[1]Chair of Computational Mathematics and Simulation Science, École Polytechnique Fédérale de Lausanne, 1015 Lausanne, Switzerland.
[2]Seminar for Applied Mathematics, ETH Zürich, 8092 Zürich, Switzerland

We present wavenumber-robust error bounds using deep neural networks to emulate the solution to the time-harmonic, sound-soft acoustic scattering problem in the exterior of a smooth, convex, two-dimensional obstacle.

The starting point of our analysis is the reduction of the scattering problem in the unbounded exterior region to its (bounded) boundary by means of the wavenumber-robust Combined Field Integral Equation (CFIE), yielding a second-kind boundary integral equation posed on the smooth surface Γ of the scatterer. This BIE is well-posed in $L^2(\Gamma)$, with explicit bounds on the continuity and stability constants that depend explicitly on the (non-dimensional) wavenumber κ.

Utilizing well-known wavenumber-explicit asymptotics of the solution to this problem, as introduced in the work of Melrose and Taylor [1], we explore the numerical approximation of the BIE using fully connected, deep feed-forward neural networks (DNNs) with the Rectified Linear Unit (ReLU) as the chosen activation function [2]. The results presented here can be straightforwardly extended to different activation functions such as the hyperbolic tangent or the Rectified Power Unit.

Through a constructive argument, we prove the existence of DNNs affording an ϵ-error in the $L^\infty(\Gamma)$-norm with a fixed and small width and a depth that increases *spectrally* with the accuracy ϵ and polynomially with respect to $\log(\kappa)$. By *spectral accuracy*, we mean that there exists $\alpha > 0$ such that for each $n \in \mathbb{N}$, there exists a constant $C_n > 0$, such that for a prescribed accuracy $\epsilon > 0$, the complexity of the DNN is bounded by $C_n \epsilon^{\frac{\alpha}{n}}$.

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