A deep neural network approach for parameterized PDEs and Bayesian inverse problems

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We consider the simulation of Bayesian statistical inverse problems governed by large-scale linear and nonlinear partial differential equations (PDEs). Markov chain Monte Carlo (MCMC) algorithms are standard techniques to solve such problems. However, MCMC techniques are computationally challenging as they require a prohibitive number of forward PDE solves. The goal of this paper is to introduce a fractional deep neural network (fDNN) based approach for the forward solves within an MCMC routine. Moreover, we discuss some approximation error estimates. We illustrate the efficiency of fDNN on inverse problems governed by nonlinear elliptic partial differential equations and the unsteady Navier-Stokes equations. In the former case, two examples are discussed, respectively depending on two and 100 parameters, with significant observed savings. The unsteady Navier-Stokes example illustrates that fDNN can outperform existing DNNs, doing a better job of capturing essential features such as vortex shedding.